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## Abstract

There is now enough observational information available to show that the interstellar magnetic field in the general neighborhood of the sun is, on the average, parallel to the plane of the galaxy, with an average strength somewhere between  $10^{-6}$  and  $10^{-5}$  gauss. This paper points out certain dynamical requirements for the existence of such a field. The paper is based on the assumption that the intergalactic medium, whatever it may be, exerts pressures on the galaxy which are small compared to  $10^{-12}$  dynes/cm<sup>2</sup>. It can then be shown that the galactic, or interstellar, magnetic field must be confined to the galaxy by the weight of the gas threaded by the field. It is shown further that the gas holding in the interstellar field must be distributed throughout the disk of the galaxy. It is then shown that the interstellar gas field system is subject to a universal Rayleigh-Taylor instability of such a nature that the interstellar gas tends to concentrate into pockets suspended in the field. The cause of the instability may be thought of as a hydromagnetic self-attraction in the interstellar gas, which may be ten times larger than the gravitational self-attraction of the gas. It is this hydromagnetic self-attraction which produces the observed tendency of the interstellar gas to be confined in discrete clouds.

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The calculations and arguments do not restrict the over-all topology or the strength of the galactic field, which must still be determined from observation apparently.

## I. Introduction

The strength and topology of the galactic magnetic field is a central problem in the origin of cosmic rays, galactic nonthermal radio emission, and the dynamics of the arms of the galaxy (see Wentzel, 1963; Woltjer, 1963, 1965; Parker, 1966a). The early optical polarization measurements of Hiltner (1949, 1951, 1956; Hall, 1949) indicate a large-scale average field parallel to the plane of the galaxy (Davis and Greenstein, 1951), but uncertainty in the composition of the interstellar dust grains responsible for the polarization prevents a quantitative estimate of the field strength from these observations (Greenberg, 1964). Both the radio observations (Morris and Berge, 1964) and the polarization of starlight (Smith, 1956; Behr, 1956) indicate that the local lines of force lie parallel to the direction of galactic longitude  $l^{\text{I}} = 70^\circ \pm 20$ , which agrees with the direction of the spiral arm determined from the distribution of interstellar gas and O associations (Weaver, 1953; van de Hulst, et al., 1954, Westerhout, 1957). Radio observations now seem to place an upper limit of about  $5 \times 10^{-6}$  gauss on the large-scale average field strength in the disk of the galaxy. Morris and Berge (1964) point out that the Faraday rotation measures indicate a reversal of the field across the plane of the galaxy. Thus, taken together, the observations seriously limit the ideas concerning the general nature of the galactic magnetic field. But so far, theory and observation, either separately or together, are unable to give a unique picture of the general galactic (interstellar) field.

It is the purpose of the present paper to point out some theoretical facts which further limit the possible magnetic configurations in the galaxy. In particular the considerations give an upper limit on the field strength and a unique dynamical

structure for the interstellar gas clouds in the galactic magnetic field.

The theoretical facts on which the arguments are based are of an elementary nature, but sometimes a rather tedious calculation is required to establish the individual fact. Hence, in order that the main thread of the argument not be repeatedly interrupted, many of the calculations are placed in the appendices with only a reference to the results of the calculation in the text.

## II. Equilibrium of Force-Free Fields

It is assumed that, apart from the general rotation of the galaxy, the galactic magnetic field (a) is in a quasi-stationary equilibrium state, (b) is limited to the galaxy and the galactic halo\*, and (c) experiences no significant inward pressure from intergalactic space. With these assumptions as a starting point, the first deduction is that the galactic magnetic field must be confined to the galaxy by the weight of the gas enmeshed in the field.

It is a simple matter to prove that a magnetic field can be confined only by the weight of the gas through which it penetrates. The appropriate virial equations are worked out in Appendix I. It is shown that the magnetic field energy must be added to the usual term  $2T$  in the virial, where  $T$  represents the internal kinetic energy of the matter. In the classical virial equation the gravitational potential energy must overcome the expansive effects of  $2T$  if a stationary equilibrium is to be achieved. Adding a magnetic field means that the

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\* Sciama (1962) has suggested that the galactic field extends throughout the Local Group. We are skeptical of the idea for the reasons given elsewhere (Parker, 1966a) but, as a matter of fact, the conclusions of the present paper would not be altered by Sciama's field configuration. The important point is that the field in intergalactic space is assumed to be small compared to the field in intergalactic space in both cases.

gravitational potential energy must overcome the expansive effects of  $2T$  plus the total magnetic energy  $\int dV B^2/8\pi$ . Magnetic fields are never self-contained. Their presence increases the tendency for the system to expand, and the expansion effect must be overcome by the weight of the gas distributed along the lines of force.

It is shown further in Appendix I that gas clouds from which the magnetic field is excluded do not contribute to confining the galactic field. The physical reason is simply that the galactic field is free to flow around the field-free gas clouds and escape from the galaxy if not confined by other forces. The only means for containing the magnetic field of the galaxy is the weight of the gas penetrated by the field.

The theoretical fact that a magnetic field can be confined to an isolated star system only by the weight of gas threaded by the field permits two distinct possibilities for the galactic magnetic field. The first possibility is that the galactic field in the disk of the galaxy, where we observe it, is held down pretty much throughout the disk by the weight of the gas there. The second possibility is that the field in the disk is not held down throughout the disk but is confined to the galaxy by gas in the galactic nucleus. To take the second case first, the field would then be largely force-free throughout the disk of the galaxy, a possibility that has been considered by a number of authors. In this case, every magnetic line of force must be tied to the galactic nucleus in order to be confined to the galaxy.\* From purely geometrical considerations it follows that the field density must increase at least as fast as  $1/r^2$

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\* It is not possible to confine lines of force which fail to pass through the nucleus of the galaxy by linking them through lines which do thread through the nucleus.

between here and the center of the galaxy. Indeed, the formal calculation of force free fields (Lüst and Schlüter, 1954; Chandrasekhar, 1956) shows that a localized field confined at the origin must increase toward the origin at least as fast as  $1/r^3$ . Hence a field of  $5 \times 10^{-6}$  gauss at a distance of 10 kpc from the galactic nucleus becomes  $5 \times 10^{-3}$  gauss at a distance of 1 kpc. So strong a field would dominate all interstellar gas motions within 5 kpc of the center of the galaxy, preventing differential rotation of the gas. The polarization effects and synchrotron radiation from the field toward the center of the galaxy would be enormous. It is our impression, therefore, that the possibility can be ruled out on the basis of observations.

Another force-free configuration that has been considered is that the galactic magnetic field is in the form of force-free twisted ropes of magnetic flux extending along the galactic arm. Such a configuration in no way avoids the general virial condition that the field must be confined by the weight of gases, but it is a different situation from that in which all lines are tied straight into the galactic nucleus. The case of a twisted rope is worked out in Appendix II, where it is shown that unless the external (intergalactic) pressure is equal to half the average internal magnetic pressure, the twisted rope will buckle because of compressive stresses along its length. The system would be so unstable as to transform itself into some other configuration within  $10^8$  years.\* The buckling *might* be stabilized by enough internal gas, of course, but the gas is then confining the field, which is not the possibility under discussion.

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\* The calculations discussed in sections IV and V show that it is not possible to stabilize the arm with a dense string of stars along the arm.

Altogether then, there is no steady magnetic configuration consistent with present observations which is force-free throughout the disk of the galaxy and is confined only in the galactic nucleus. The alternative is to assume that the galactic field in the disk is contained more or less throughout the disk by the weight of the interstellar gas in the disk. The next section works out some of the consequences of this.

### III. Equilibrium of a Field Confined to the Disk

If we conclude that the galactic field must be contained by the weight of the interstellar gas throughout the disk, the immediate question is what this containment requires in the way of an average interstellar gas density. Either the tensor virial equations or the hydrostatic pressure equation may be used to treat the problem. To see what is needed, consider the simple case suggested by the polarization observations, that the field in the disk is largely parallel to the disk. Then if the field density is the function  $B(z)$  of distance measured perpendicular to the plane of the disk, the condition for quasi-static support of the interstellar gas density  $\rho(z)$  against the gravitational acceleration perpendicular to the plane of the galaxy is

$$\frac{d}{dz} \left( p + P + \frac{B^2}{8\pi} \right) = -\rho(z) g(z) \quad (1)$$

where  $p(z)$  is the gas pressure and  $P(z)$  is the cosmic ray pressure. In the simplest case suppose that the three pressures are all proportional, with

$$B^2/8\pi = \alpha \rho, \quad P = \beta \rho \quad (2)$$

where  $\alpha$  and  $\beta$  are dimensionless constants. The possible variations of  $\alpha$  and  $\beta$  with  $z$  do not alter the conclusions. The scale height  $\Lambda$  of the gas-field system is

$$\frac{1}{\Lambda} \equiv \left| \frac{1}{\rho} \frac{d\rho}{dz} \right|.$$

Writing  $v^2$  for the total rms velocity of the gas in the  $z$ -direction including cloud motions, we have  $\rho = \rho v^2$  and

$$\Lambda = \frac{v^2}{g} (1 + \alpha + \beta) \quad (3)$$

This scale height is to be compared with the scale height  $\lambda$  of the stars in the same gravitational field

$$\lambda = \frac{u^2}{g} \quad (4)$$

where  $u(z)$  is the rms velocity of the stars in the  $z$ -direction. It follows that

$$\frac{\Lambda u^2}{\lambda v^2} = 1 + \alpha + \beta. \quad (5)$$

The scale height of the interstellar gas is known to be about 100 pc (Schmidt, 1956; van de Hulst, 1958; Rougoor, 1964). The scale for the distribution of late type stars, such as  $K$  - giants is about 300 pc (Oort, 1959; Hill, 1960). The rms velocity of the observed gas clouds in the direction perpendicular to the plane of the galaxy is about 8 km/sec\* (see discussion in Gould, et al., 1963). The rms velocity of late stars in the direction perpendicular to the plane of the galaxy is 15 - 20 km/sec: The observational numbers are 18.6 km/sec for the dwarf G-stars in the neighborhood of the sun and 15.6 km/sec for late stars brighter than magnitude 6 (See Trumpler and Weaver, 1962; Charlier, 1926). A good median value, therefore, might be 17 km/sec. It follows from these observational values that  $\Lambda v^2 / \lambda v^2 \approx 1.5$ .\*\* Eqn. (5) permits the immediate calculation of  $\alpha + \beta$ , giving  $\alpha + \beta \approx 0.5$  for the numbers adopted.

We are interested principally in the number density  $N(z)$  of the interstellar gas, so (5) is rewritten as

$$N = \frac{P + B^2 / 8\pi}{M v^2 (\Lambda v^2 / \lambda v^2 - 1)} \quad (6)$$

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\* The high velocity clouds at high galactic latitude (Munch and Zirin, 1961; Muller, et al. 1963) are not included in this number. Doing so would increase the large gas density calculated from (6)

\*\* An observational value of  $\Lambda v^2 / \lambda v^2 < 1_2$  would imply an intergalactic pressure of the order of  $10^{-12}$  dynes/cm<sup>2</sup>.

The same result can be deduced from the tensor virial equations (Appendix I) if the reader prefers.

The cosmic ray pressure  $P$  is presently observed to be about  $0.5 \times 10^{-12}$  dynes/cm<sup>2</sup> (Parker, 1966b). There is now good evidence that the present cosmic ray intensity is typical of conditions at the solar system for the past  $10^9$  years (Zahringer, 1964; Anders, 1965) and hence is typical for much of the galaxy (see discussion in Parker, 1966a). A Magnetic field of  $5 \times 10^{-6}$  gauss gives a pressure of  $B^2/8\pi = 1 \times 10^{-12}$  dynes/cm<sup>2</sup>, yielding  $N = 2.8$  hydrogen atoms/cm<sup>3</sup> to account for the observed small scale height of the gas. This density is rather larger than the usually estimated 1 atom/cm<sup>3</sup>. The necessary gas density can be reduced to 1/cm<sup>3</sup> only by making  $B \leq 2 \times 10^{-6}$  gauss so that  $B^2/8\pi \ll P$ .

The observational value of  $\Lambda v^2/\lambda v^2$  suggests, therefore, that the interstellar hydrogen density must be 3/cm<sup>3</sup> if the galactic field is as strong as  $5 \times 10^{-6}$  gauss. A field much stronger than  $5 \times 10^{-6}$  gauss would require much greater interstellar gas densities for confinement, which would seem to put an upper limit on the field of  $10 \times 10^{-6}$  gauss or less. Thus the theoretical necessity for a sufficient interstellar gas density limits the field at about the same level, viz  $5 \times 10^{-6}$  gauss, as the present radio observations of the galactic field.

It is interesting to note that there are other reasons besides the dynamics of the gas-field system for believing that the average interstellar gas density is rather more than the observed atomic hydrogen density. For instance Gould, et al. (1963) point out that the gravitational acceleration  $g(z)$  perpendicular to

the plane of the galaxy can be deduced from the observed scale height and velocity of the K-giants. The value of  $g$  so obtained requires a larger mass in the disk of the galaxy than the stars plus  $1 \text{ atom/cm}^3$ . They suggest that the additional mass is interstellar gas with a density of  $5 \text{ atoms/cm}^3$ ,  $4 \text{ atoms/cm}^3$  being molecular rather than atomic hydrogen.

Recent work by Toomre (1966) and Julian and Toomre (1966) indicates that an average interstellar density of  $3/\text{cm}^3$  would make the galactic arms understandable in terms of gravitational forces alone (see also Lin and Shu, 1964).

The question of the total interstellar gas density is of central importance in calculating the rate at which cosmic rays are generated in the galaxy (see discussion in Ginsburg and Syrovatskii, 1964; Parker, 1966a). The calculated rate of generation varies directly with the mean interstellar density and is of the order of  $10^{41} \text{ ergs/sec}$  with  $1 \text{ atom/cm}^3$ , approaching the total energy output of all the novae and supernovae in the galaxy.

It is to be hoped that the interstellar molecular hydrogen can soon be looked for. The importance for further qualitative discussion of the galactic gas-field system is obvious. Fortunately the qualitative arguments presented in this paper do not depend on the precise values of either the gas or the field density.

#### IV. Stability of a Field Confined to the Disk

The arguments presented up to this point have shown that the galactic magnetic field in the disk must be contained by the weight of the gas in the disk. The polarization of starlight suggests that the average field in the disk is parallel to the plane of the disk, and a particularly simple example of such equilibrium was

considered in the previous section. The next question concerns the stability of such an equilibrium. The field confined by the weight of the gas is quite different from the laboratory plasma confinement with which we are familiar, where the field confines the gas. So it is necessary to look into the matter rather carefully. We begin with the example employed in the previous section, of a magnetic field of density  $B(z)$  in the horizontal  $y$ -direction. The gravitational acceleration  $g$  is in the negative  $z$ -direction and the thermal gas density  $\rho$  in the field is supported against gravity by the magnetic field, the thermal gas pressure, and the cosmic ray gas pressure.

We consider first the simple convective interchange of the magnetic lines of force, as sketched in Fig. 1. The stability criterion is worked out in Appendix III. What information is available on the interstellar medium suggests that the system may be weakly unstable. If shearing is present, in which the field is parallel to the plane of the galaxy but the direction of the field rotates about a vertical axis with <sup>increasing</sup> height above the plane, the interchange mode is probably stable. Shearing is a well known laboratory procedure to eliminate interchange instability in the magnetically confined plasma.

It is more interesting to consider the stability of the system against transverse waves in the magnetic field. This calculation is made in Appendix IV. for an isothermal atmosphere with the pressures of the thermal gas, the magnetic field, and the cosmic ray gas in the constant ratio  $1 : \alpha : \beta$  (see eqn. (2)). The atmosphere is in a constant gravitational field  $g$  in the negative  $z$ -direction and self gravitation is neglected. The pressure and density of

the thermal gas are related by the simple equation of state  $\delta p/p = \gamma \delta \rho/\rho$  where  $\gamma$  is a constant. The calculations consider a perturbation with a periodic variation expiky along the large-scale magnetic field. Requiring that the perturbation vanish at the "base" of the atmosphere, say at  $z = 0$ , and remain finite at  $z = +\infty$  leads to instability whenever

$$\gamma - 1 < \frac{\alpha/2 + \beta + (\alpha + \beta)^2}{(1 + 3\alpha/2 + \beta)} \quad (7)$$

The thermal gas by itself would, of course, be stable provided only that  $\gamma > 1$ . The horizontal magnetic field and the cosmic ray gas both drive the system toward instability, so that  $\gamma$  must exceed 1 by the amount indicated in (7) if the thermal gas is to maintain stability. The equilibrium conditions 3 atoms/cm<sup>2</sup>,  $B = 5 \times 10^{-6}$  gauss,  $P = 0.45 \times 10^{-12}$  dynes/cm<sup>2</sup> give instability for any  $\gamma$  less than 1.31; the conditions 1 atom/cm<sup>3</sup>,  $B \lesssim 2 \times 10^{-6}$  gauss,  $P = 0.45 \times 10^{-12}$  dynes/cm<sup>3</sup> give instability for any  $\gamma < 1.45$ .

As pointed out near the end of Appendix III, radiative transfer in the interstellar medium is so effective that for perturbations with periods of the order of  $10^7 - 10^8$  years, such as we are considering here, a density increase in the thermal gas produces little change in the temperature. If there is any effect at all, it is probably for the temperature to decline with increasing density

( $\gamma < 1$ ) . So put  $\gamma \approx 1$  as a conservative estimate (see discussion in Parker, 1953). The thermal gas is then only marginally stable by itself. The magnetic field and the cosmic ray gas make the total gas-field system unstable.

- 4 The calculations show that the growth time is typically  $3 \times 10^7$  years. This time is short compared to the life of the galaxy, the time of formation of the galactic arms, and the time in which the thermal gas condenses into stars. So the instability appears to be dynamically important for the state of the interstellar gas-field system.

Let us inquire into the nature of the instability. Neither the magnetic field nor the cosmic ray gas is subject to gravity, so in equilibrium they must be confined by the weight of the thermal gas. That is to say, some portion of the weight of the thermal gas is supported by the magnetic field and cosmic ray pressure. So if a perturbation is introduced involving vertical displacement of some portion of the horizontal equilibrium field, the thermal gas tends to slide downward along the magnetic lines of force away from the raised portion of the field into the lower regions along the lines of force. This diminishes the overburden on the raised portion, permitting the field and cosmic ray gas to expand upward there, causing further slipping of the thermal gas downward along the lines of force, etc. At the same time that the raised portions of the field are being unloaded in this manner, the burden on the lower portions is being increased. Only if  $\gamma$  is sufficiently greater than one will the thermal gas resist the tendency to slide downward along the field and so give a stable atmosphere.

It is evident that the instability is distinct from the well known Jean's gravitational instability, which is the result of self-gravitation. The instability is also distinct from the lack of equilibrium caused by unlimited cosmic ray inflation of the fields at the surface or the galactic disk (Parker, 1965). The instability is related to the familiar Rayleigh-Taylor instability in which a dense fluid supported from beneath by the pressure of a light fluid tends to drip downward through the

light fluid.

One may inquire if the instability may be avoided with some magnetic field configuration other than the simple horizontal field. Consider, for instance, a circular geometry, representing a cross-section of the interstellar gas in a self-gravitating galactic arm. Wrap the magnetic lines of force around the arm so that the tension in the field might stabilize the configuration. The stability of the system is treated in Appendix V. The calculations show that the system is as unstable as the horizontal field. A twisted rope of magnetic field, involving lines of force both along and around the arm, fares no better. The flat and circular geometries considered in the Appendices by no means exhaust all the possibilities, of course. For instance, the tidal forces exerted on any one galactic arm by the rest of the galaxy would distort a circular geometry into an elliptical one. Or the field may be twisted into many small parallel ropes. But nothing essentially new is added to the problem by such complications. The basic point established by the examples given here is that if the lines of force of a large-scale field along the galactic disk or arm are confined by the weight of the thermal gas, then the gas always tends to drain downward along the magnetic lines of force into the lowest region along each line. The instability is unavoidable unless the thermal gas is strongly stable by itself. The interstellar thermal gas is not significantly stable by itself

$(\gamma \lesssim 1)$  , so the magnetic field and the cosmic ray gas drive the interstellar gas field system unstable in periods of  $10^7 - 10^8$  years.

## V. The Long-Term State of the Interstellar Gas-Field System

It has been demonstrated that a large-scale equilibrium interstellar magnetic field (suggested by present magnetic observations) is intrinsically unstable in a short time, of only  $3 \times 10^7$  years. The instability must quickly destroy the equilibrium. The question is, then, what is the dynamical state of the interstellar magnetic field now, after  $10^9 - 10^{10}$  years? The answer to this question follows from the nature of the instability. (Some examples are worked out in detail in Appendix IV.) The instability is the result of the thermal gas draining down along the magnetic lines of force into the low regions along the field, thereby burdening down the low regions and releasing the field between the low regions to expand upward. A sketch of the resulting field configuration along a line of force is shown in Fig. 2. The horizontal spacing of the gas pockets in the low regions is of the general order of magnitude of the scale height of the system (see Appendix IV). It must be concluded from the calculations that, if there is a large-scale interstellar field confined to the galaxy, then the interstellar gas confining the field is presently suspended in the field in discrete clouds with separations of the order of 100 pc, i.e.  $(10 - 10^3)$  pc. The field has the general arched configuration shown in Fig. 2.

It is interesting that the dynamical properties of a large-scale magnetic field should lead to this conclusion, because the conclusion may help resolve a perplexing problem concerning the maintenance of some of the less massive interstellar gas clouds. It is observed (Adams, 1949; Münch and Unsöld, 1962) that the interstellar gas exists mainly in widely separated discrete clouds

(see the recent high resolution observation of interstellar absorption lines by Livingston and Lynds, 1964). The usual explanation for the discrete character of the interstellar gas is self-gravitation of the individual clouds, but there is the problem that in many cases the cloud masses inferred from the observations do not seem to be large enough to maintain the cloud in equilibrium by self-gravitation alone (see, for instance Kahn and Dyson, 1965). For instance the self-gravitation of a spherical cloud with a diameter of 20 pc and a density of 10 hydrogen atoms/cm<sup>3</sup> can hold the cloud together only if the internal motions are 0.7 km/sec. A higher density of 100 atoms/cm<sup>3</sup> can contain internal motions of only 2.2 km/sec. But even the thermal velocities are this large, to say nothing of the 10 km/sec motions expected from collisions between clouds and from the passage of hot luminous stars through the region. So there is some question as to the means by which the apparent identity of the smaller, more tenuous, interstellar gas clouds is maintained. The new point arising in the galactic field configuration presented in this paper is that the self-gravitation of the individual gas clouds is supplemented, in the configuration shown in Fig. 2, by the gravitational field of the galaxy as a whole.

To illustrate the supplement to self-gravitation in a direct way, and to establish that the supplement may be large in many cases, consider two parallel infinitely long slender cylinders of gas lying across the horizontal magnetic field  $B_z$  and supported by the magnetic field in the large-scale gravitational field  $g_z$ . If  $m$  is the mass per unit length, the current  $I$  in each cylinder is

$$I B_0 / c = m g.$$

It is well known that two parallel currents attract each other with the force

$$F = \frac{2 I^2}{c^2 s} = \frac{2}{s} \frac{m^2 g^2}{B_0^2}$$

where  $s$  is the distance between the two cylinders. Note that this force of attraction is proportional to the square of the masses and inversely proportional to the distance  $s$ , just as the gravitational force

$$F_G = \frac{2 G m^2}{s}.$$

It follows that the ratio of the pseudo-self-gravitation to the gravitation is

$$\frac{F}{F_G} = \frac{g^2}{G B_0^2},$$

which may be extremely large in regions of weak galactic magnetic field  $B_0$  and strong galactic gravitational field  $g$ .

The general point is that the currents

$$\underline{j} = c \nabla \times \underline{B} / 4\pi$$

which confine the large-scale field to the galaxy are locally self-attracting.

The self-attraction of the currents leads to the instability pointed out in

section IV and to the pseudo-self-gravitation pointed out in the present section. The pseudo-self-gravitation has the same form as gravitation for parallel cylinders, but is somewhat more complicated for structures which do not stretch all the way across the field. The simple example of two short segments of length  $\delta$  supported by the field is easily worked out, showing how the situation is complicated by currents flowing along the magnetic lines of force, in addition to those across.

The gravitational acceleration perpendicular to the disk of the galaxy is estimated (see for instance Oort, 1960; Hill, 1960; Gould, et al., 1963) to be of the order of  $5 \times 10^{-9} \text{ cm/sec}^2$  at a distance of 100 pc above the central plane of the disk of the galaxy. A field of  $B = 5 \times 10^{-6} \text{ gauss}$  then gives a ratio  $F/F_G = 15$ . It is evident from this example that the effective attraction between two elements of gas may be enormously increased above self-gravitation and may, therefore, be an important effect in confining the interstellar gas to discrete clouds.

It is evident the attraction should be included in calculations of Jean's instability criterion. To a first approximation the effect may be represented by a suitable increase in the effective gravitational constant  $G$ . Hence, for a given average gas density the result is a smaller mass for the individual contracting cells of gas.

It should be noted that the attraction vanishes on the central plane of the disk of the galaxy because the component  $g$  of the gravitational field perpendicular to the plane of the disk vanishes there. It follows, therefore,

that there should be some tendency for small tenuous gas clouds (which have little self-gravitation) to be defined more sharply in regions removed from the central plane than in regions near the central plane. It suggests, too, that there may be a tendency for stars of larger mass to be formed near the central plane of the disk.

## VI. Discussion

The arguments of the preceding sections have passed rather directly to the conclusions, leaving a number of important points untouched. This section is intended to go back and pick up some of these points.

First of all, it should be noted that the calculations and ~~conclusions~~ presented in this paper do not restrict the overall magnetic configuration of the galactic magnetic field. The calculations have to do only with the small-scale ( $10 - 10^3$  pc) properties of the galactic field. They apply to a twisted rope of magnetic flux along a galactic arm as well as to a large-scale horizontal field in the disk, etc.

The arguments began with the idea of a large-scale field of some sort with a tendency to lie parallel to the disk of the galaxy because the observed polarization of the light of distant stars seems to require this. The question is, then, whether the final pendulant configuration (Fig. 2) is consistent with this starting point of view. We suggest that it is. The space average of the vertical component of the magnetic field is close to zero in any simple gas cloud, and in the intercloud region as well. The light path for the significantly reddened stars, in which the polarization is observed, usually passes through more than one gas

cloud (see again Livingston and Lynds, 1964). Presumably, therefore, the net direction of polarization of the light of most reddened stars comes close to the overall average field in the disk, which the observations (Hiltner, 1949, 1951, 1956) show (Davis and Greenstein, 1951) is close to the plane of the galaxy. It must be remembered that the averaging is generally believed to be sufficient to obscure even a possible overall twisting of magnetic lines of force along the galactic arm.

The velocities of the individual interstellar gas clouds relative to the local galactic rotation are statistically isotropic so far as observations can tell. And, so far as observations can tell, the magnitude of the random velocities does not vary with height above the plane of the galaxy. The interstellar cloud system sketched in Fig. 2 seems to fit this picture fairly well. If the motion of each cloud can be considered a harmonic oscillation about some equilibrium position, then random excitation of the oscillator leads to a statistically isotropic velocity no matter how weak the binding may be in one direction and how strong in another.\* Since the gas clouds are all tied into the same large-scale magnetic field system, it is not surprising if the degree of excitation should be independent of distance from the galactic plane.

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\* The displacement amplitudes are not isotropic in an anisotropic oscillator.

## Appendix I. The Virial Equations for the Gas-Field System

The arguments presented in section III are based on the result, easily shown from the virial equations, that a galactic magnetic field can be confined to the galaxy only by the weight of the gas which it penetrates.

Consider the virial equation for the interstellar gas and magnetic field system. Let  $\rho$  represent the gas density and  $B_i$  the field. The Maxwell stress tensor is

$$M_{ij} = -\delta_{ij} \frac{B^2}{8\pi} + \frac{B_i B_j}{4\pi}. \quad (I1)$$

Let  $\phi$  represent the total gravitational potential, due to stars and gas together.

Then the motion of each atom is given by

$$\rho \frac{d^2 x_i}{dt^2} = \frac{\partial M_{ik}}{\partial x_k} - \rho \frac{\partial \phi}{\partial x_i}. \quad (I2)$$

Multiply by  $x_j$  and add the result to the same equation with  $i$  and  $j$  interchanged. The usual manipulation of  $x_i d^2 x_i / dt^2$ , together with integration by parts, leads to

$$\begin{aligned} \frac{d^2 I_{ij}}{dt^2} = & 4T_{ij} + \int dS_k (x_i M_{jk} + x_j M_{ik}) - 2 \int dV M_{ij} \\ & - \int dV \rho \left( x_i \frac{\partial \phi}{\partial x_i} + x_j \frac{\partial \phi}{\partial x_j} \right) \end{aligned} \quad (I3)$$

where  $I_{ij}$  is the moment of inertia tensor and  $T_{ij}$  is the kinetic tensor

$$I_{ij} = \int dV \rho x_i x_j, \quad T_{ij} = \frac{1}{2} \int dV \rho \frac{dx_i}{dt} \frac{dx_j}{dt}. \quad (14)$$

It will be assumed that there are no external forces exerted on the gas-field system, so the surface integral vanishes when calculated over a sufficiently distant external enclosing surface.

So far the calculation is nothing more than the standard derivation of the virial equation (see for instance Chandrasekhar and Fermi, 1953; Parker, 1954; Chandrasekhar, 1961). If the gravitational potential  $\phi$  were the result only of the gas density  $\rho$  ( $\nabla^2 \phi = 4\pi G\rho$ ), it would be possible to write the last term on the right hand side of (13) in terms of the total gravitational potential  $\Phi = \int dV \rho \phi$  in the usual way. But since other matter, such as the stars, contribute to  $\phi$ , this is not possible. The precise value of the integral depends now on the spatial form of  $\phi$ . To pick a simple example suppose that the gas-field system is in a spherically symmetric parabolic potential well,

$$\phi(r) = g \frac{r^2}{2a} \quad (15)$$

where  $g$  is the inward gravitational acceleration at the characteristic radial distance  $r = a$ . Then it follows that

$$\int dV \rho \left( x_i \frac{\partial \phi}{\partial x_j} + x_j \frac{\partial \phi}{\partial x_i} \right) = \frac{2g}{a} \int dV \rho x_i x_j$$

$$= \frac{2g}{a} I_{ij} \quad (16)$$

and the trace of this is

$$\frac{2g}{a} I_{ii} = \int dV \rho 2x_i \frac{\partial \phi}{\partial x_i} = 4 \int dV \rho \phi$$

$$\equiv 4\Phi \quad (17)$$

Thus, if there are no forces on the external surface of the system, (13) reduces to

$$\frac{d^2 I_{ij}}{dt^2} + \frac{2g}{a} I_{ij} = 4T_{ij} - 2 \int dV M_{ij} \quad (18)$$

Contracting on the indices gives

$$2\Phi = 2T + \int dV B^2 / 8\pi \quad (19)$$

for equilibrium, where  $T$  is the total kinetic energy of the internal motions and

$\int dV B^2 / 8\pi$  is recognizable as the total magnetic energy of the system. The gravitational potential <sup>energy</sup>  $\Phi$  here is measured above the potential at the origin, rather than below the potential at infinity. Equation (19) illustrates

the fact that the kinetic energy and the magnetic energy must be contained by a comparable amount of gravitational potential energy. A magnetic field is not self containing. It must be confined by the weight of the gas which it threads.

There has been some discussion of the possibility of interstellar gas clouds from which the galactic magnetic field is excluded for one reason or another (see Woltjer, 1961). The diamagnetic clouds lie outside the gas-field system with which we are concerned, but they exert forces on the gas field system at their surfaces. It is necessary to work out the surface integrals. The contracted form  $\int dS_i x_i M_{ij}$  is sufficient, yielding  $\int dS_i x_i B^2 / 8\pi$  from (II) and the fact that  $dS_i B_i = 0$  for clouds which exclude the external field. To make the problem tractable, suppose that the diamagnetic clouds are spherical with radius  $R$ , widely separated, and small compared to the scale  $L$  of the external field. Consider a cloud with its center at  $x_i = X_i$ . Let the field in that neighborhood be  $e_i B(X_i)$  before the cloud was introduced, where  $e_i$  is the unit vector. Following the introduction of the cloud the field in that neighborhood becomes  $-\partial\psi/\partial x_i$  with

$$\psi = B(X_i) \left[ \left(1 + \frac{R^3}{2r^3}\right) \xi_i e_i + O\left(\frac{R}{L}\right) \right], \quad (110)$$

where  $\xi_i$  represents rectangular coordinates measured from the center of the sphere and  $r$  is the radial distance  $(\xi_i \xi_i)^{1/2}$  from the center. It is readily shown that the work  $W$  required to inflate the sphere to a radius  $R$  against the pressure exerted on it by the external field is

$$W = \frac{3}{2} \left( \frac{4\pi R^3}{3} \right) \frac{B^2(X_i)}{8\pi} = \frac{1}{4} B^2(X_i) R^3. \quad (I11)$$

The external field  $B(X_i)$  is not uniform, of course, so there will be a net bouyant force (Parker, 1955, 1957) exerted on the cloud. The force  $F_i$  is

$$F_i = - \frac{\partial W}{\partial x_i}, \quad (I12)$$

where the differentiation is carried out with  $R$  fixed.

To evaluate the surface integral write  $x_i = X_i + \xi_i$ . Then, for each diamagnetic cloud,

$$\int dS_i x_i \frac{B^2}{8\pi} = X_i \int dS_i \frac{B^2}{8\pi} + \int dS_i \xi_i \frac{B^2}{8\pi}$$

The first integral on the right hand side is just the negative of the total force exerted on the cloud by the external magnetic field. The second integral is easily shown to be equal to  $3W$ , since  $dS_i \xi_i = 2\pi R^3 \sin\theta d\theta$  and  $B^2 = 9 B^2(X_i) \sin^2\theta / 32\pi$ , where  $\cos\theta = e_i \xi_i / r$ . Hence

$$\begin{aligned} \int dS_i x_i \frac{B^2}{8\pi} &= -X_i F_i + 3W \\ &= X_i \frac{\partial W}{\partial X_i} + 3W \\ &= \frac{\partial}{\partial X_i} (W X_i) \end{aligned} \quad (I13)$$

It follows that the scalar virial equation for the gas-field system external to a number of diamagnetic spheres is

$$2\Phi = 2T + \int dV \frac{B^2}{8\pi} + \sum \frac{\partial}{\partial x_i} W x_i \quad (\text{I14})$$

in place of (I9). The sum is over all the spheres. The sum can be greatly simplified if we consider a very large number of small diamagnetic spheres uniformly distributed over the cloud. If  $\gamma$  is the number of clouds per unit volume, then  $\sum$  can be replaced by  $\int dV \gamma$  and

$$\begin{aligned} \sum \frac{\partial}{\partial x_i} W x_i &= \gamma \int dV \frac{\partial}{\partial x_i} (x_i W) \\ &= \gamma \int dS_i x_i W \end{aligned}$$

where the surface integral is over the external surface of the gas-field system. But the field, and hence  $W$ , vanish at large distance from the gas-field system. So the surface integral vanishes and (I14) reduces to the same form as the virial equation (I9) for the system without diamagnetic clouds. The diamagnetic clouds do not confine the field, which can be contained only by a comparable gravitational potential.

## Appendix II. Forces in a Twisted Rope of Magnetic Field

A few remarks are required concerning the stresses in a force-free twisted rope of flux, sometimes considered as a possible configuration for the galactic arm field. The magnetic field in an axially symmetric force-free tube of flux can be expressed (Schlüter, et al., 1953) in terms of a generating function  $F(\varpi)$  where  $\varpi$  is distance measured from the axis of the tube, along which distance is measured by  $z$  and around which the azimuthal angle is  $\phi$ . The components of the magnetic field can always be written as

$$B_\phi^2 = -\frac{1}{2} \varpi \frac{dF}{d\varpi}$$

$$B_z^2 = F(\varpi) + \frac{1}{2} \varpi \frac{dF}{d\varpi},$$

so that the arbitrary generating function is just the square of the magnitude of the field. The total tension  $Q$  in the rope, from  $\varpi = 0$  out to  $\varpi = a$ , is

$$\begin{aligned} Q(a) &= 2\pi \int_0^a d\varpi \varpi \left( \frac{B_z^2 - B_\phi^2}{8\pi} \right) \\ &= \frac{1}{4} \int_0^a d\varpi \varpi \frac{d}{d\varpi} (\varpi F) \\ &= 2\pi a^2 \left\{ \frac{B^2(a)}{8\pi} - \frac{1}{2\pi a^2} \left[ 2\pi \int_0^a d\varpi \varpi \frac{B^2}{8\pi} \right] \right\} \quad (\text{II}) \end{aligned}$$

after integration by parts. The tube of force can be stable only if  $Q > 0$ , for if  $Q < 0$  the tube is under compression and will buckle. Suppose that the rope terminates at some finite radius  $\varpi = a$ . There must then be an external

pressure equal to  $B^2(a)/8\pi$  exerted on the field at  $\varpi = a$ , of course.

Then it may be seen from (II) that  $Q(a) > 0$  for stability requires that the

external pressure must equal or exceed one half the average magnetic energy

density within  $\varpi = a$ . So any such force-free tube must be contained

by an external pressure comparable to  $B^2/8\pi$  within the tube. This is,

of course, nothing more than the virial condition again, stating that a magnetic

field will expand to infinity unless confined by comparable inward forces.

## Appendix III. Stability Against Convective Interchange

Consider the stability of an atmosphere  $p(z)$  with a horizontal magnetic field  $B(z)$  against interchange of the magnetic lines of force. Assume that the perturbation is independent of distance  $y$  measured along the magnetic field so that the motion is the simple  $xy$  convective interchange illustrated in Fig. 1. The magnetic field is essentially a massless fluid. The cosmic rays are another essentially massless fluid. It is assumed that the cosmic ray gas maintains its statistical isotropy, during slow compression and expansion, as a consequence of small scale irregularities in the magnetic field and/or its own internal micro-instabilities.

Suppose, then, that the thermal gas is composed of  $N(z)$  atoms, each of mass  $M$ , per unit volume. In the unperturbed state suppose that the thermal gas is polytropic

$$p(z) = p(0) \left[ \frac{N(z)}{N(0)} \right]^{\Gamma}$$

where  $\Gamma$  is a constant. Under a small perturbation assume that the pressure in a given element of volume varies as

$$\frac{\delta p}{p} = \gamma \frac{\delta N}{N},$$

in which  $\gamma$  is not necessarily equal to  $\Gamma$ . In addition to this thermal gas, suppose that there are  $n$  massless media present with pressures

$p_i$  which are all constrained to move with the thermal gas. In the equilibrium state

$$p_i(z) = p_i(0) \left[ \frac{N(z)}{N(0)} \right]^{\Gamma_i}$$

and in a perturbation

$$\frac{\delta p_i}{p_i} = \gamma_i \frac{\delta N}{N}$$

The hydrostatic equilibrium is given by

$$\frac{1}{N} \frac{dN}{dz} = - \frac{NMg}{\sum_{i=0}^n \Gamma_i p_i} \equiv -S$$

where  $S$  is a reciprocal length and the thermal gas is included in the sum as the term  $i = 0$ .

The most direct way to get at the interchange stability of this composite polytropic atmosphere is to interchange the fluid in two slender cylinders with cross-sections  $A_1$  and  $A_2$ , and compute the total work necessary to effect the interchange. The mass per unit length in the cylinder  $A_1$  is  $A_1 N(z_1) M$ , where  $z_1$  is the vertical position of  $A_1$ . The total work required to put the gas from  $A_1$  into  $A_2$  is

$$W_{12} = A_1 \left\{ N(z_1) M g (z_2 - z_1) + \sum \frac{p_i(z_1)}{\gamma_i - 1} \left[ \left( \frac{A_1}{A_2} \right)^{\gamma_i - 1} - 1 \right] \right\}.$$

The work  $W_{21}$  required to place the gas from  $A_2$  into  $A_1$  is obtained by interchanging the subscripts 1 and 2.

If  $z_2 - z_1$  is taken to be small compared to the scale height of the atmosphere, i.e. if  $S(z_2 - z_1) \ll 1$ , put  $z_2 - z_1 = \Delta z$  and write

$$\rho(z_2) = \rho(z_1) (1 - \Gamma S \Delta z + \dots),$$

$$N(z_2) = N(z_1) (1 - S \Delta z + \dots).$$

Let  $A_2$  be only a little different from  $A_1$  and express this difference in units of  $\Delta z$  so that

$$A_2 = A_1 (1 + k \Delta z).$$

The total work done  $W = W_{12} + W_{21}$  in carrying out the interchange can then be written

$$W \cong A_1 (\Delta z)^2 \left\{ N(z_1) M g (S - k) + \sum_{i=0}^n \rho_i(z_1) k [\gamma_i k - \Gamma_i S] \right\}$$

neglecting all terms  $O[(\Delta z)^3]$  and higher.

The composite atmosphere is unstable if there exists a ratio  $A_1/A_2$  i.e. a value of  $k$ , for which  $W$  is negative. If  $W$  is positive

for all  $k$ , the atmosphere is stable. Note that  $W$  becomes large without limit as  $|k|$  becomes large. Obviously there is a minimum value of  $W$  for some finite value of  $k$ . It is readily shown that the minimum occurs for  $k = S$ . The minimum value of  $W$  is

$$W_{\min} = S^2 A_1 (\Delta z)^2 \sum_{i=1}^n p_i (\gamma_i - \Gamma_i).$$

The atmosphere is unstable if the sum  $\sum p_i (\gamma_i - \Gamma_i) < 0$  and stable if the sum is greater than zero. The result is not unexpected. It is merely the statement that the atmosphere is unstable to convective interchange if the effective equilibrium temperature gradient is steeper than the effective adiabatic gradient. For a composite polytropic atmosphere the requirement for instability is that the effective equilibrium polytrope index  $\langle \gamma \rangle$  exceed the effective perturbation index  $\langle \Gamma \rangle$ , where

$$\langle \gamma \rangle \equiv \frac{\sum p_i \gamma_i}{\sum p_i}, \quad \langle \Gamma \rangle = \frac{\sum p_i \Gamma_i}{\sum p_i}.$$

Consider the stability of the interstellar medium. The temperature of the thermal gas is, so far as anyone can tell, more or less independent of height  $z$  above the plane of the galaxy, so put  $\Gamma_0 \cong 1$ . Compressing the gas at the slow rates involved here ( $\sim 10^8$  years) probably tends to lower the temperature, if anything, by virtue of the greater ability to radiate (Parker, 1953) when at high density. Presumably therefore  $\gamma_0 \leq 1$ . Hence  $\gamma_0 - \Gamma_0$

is either close to zero or negative, and the thermal gas shows no marked stability against convection and may well be actively unstable by itself.

Not enough is known about the cosmic ray gas to give definite figures for its effective  $\gamma$  and  $\Gamma$ , but its high mobility suggests that it is always near an equilibrium in any slow deformation, so that  $\gamma_1 \cong \Gamma_1$ .

Displacements of the magnetic field give  $\gamma_2 = 2$ .

There is no way of knowing  $\Gamma_2$  for the galactic field. If the field pressure is proportional to the gas pressure, then  $\Gamma_2 \cong 1$ , and the field is stable by itself. On the other hand, if there has been a significant amount of convection, perhaps forced by accretion of intergalactic gas, by the observed motions of the interstellar medium, by the nonuniform rotation of the galaxy, etc., then  $\Gamma_2$  may be close to two and the field only weakly stable.

Altogether the system appears rather neutral to convective interchange. It would be strongly stable, of course, if the thermal gas had a  $\gamma$  more nearly equal to the adiabatic value of  $5/3$ , but radiative transfer is so effective that this is probably not achieved. One would expect that the system might be stabilized against interchange by introducing shear into the magnetic field, as is done in laboratory plasma devices. The convective turnover must then lengthen the magnetic lines of force, which resist lengthening with their tension  $B^2/4\pi$ . But this introduces a new kind of instability, taken up in the next Appendix.

## Appendix IV. Stability Against Transverse Waves

Consider the stability of a composite atmosphere of thermal gas, magnetic field  $\underline{e}$ ,  $B(z)$ , and cosmic ray gas against transverse waves propagating along the magnetic field. To make the calculation tractable suppose that the gravitational acceleration  $g$  and the thermal gas temperature  $T$  are independent of  $x, y, z$ . It will be found convenient to introduce the thermal velocity  $u = (kT/M)^{1/2}$ . Then the pressure and density are related by  $p(z) = u^2 \rho(z)$ . It will be assumed, as in Appendix III, that the cosmic ray gas maintains its statistical isotropy while flowing along the magnetic lines of force in the slow ( $10^7$  years) distortions of the field to be considered here. Suppose that the magnetic and cosmic ray pressures are confined by the weight of the thermal gas and are simply proportional to the thermal gas pressure at each point, so that eqn. (2) in the text may be applied. Hydrostatic equilibrium leads to

$$\frac{1}{P} \frac{dP}{dz} = - \frac{g}{u^2(1+\alpha+\beta)} = \frac{1}{\rho} \frac{d\rho}{dz} = \frac{1}{P} \frac{dP}{dz} = \frac{2}{B} \frac{dB}{dz} = - \frac{1}{L}, \quad (\text{IV1})$$

where  $L$  is the scale height of the atmosphere. Introduce a perturbation

$\exp i\omega t$  with wave vector parallel to the  $yz$ -plane\* involving the velocity components  $v_y$  and  $v_z$ . Express the magnetic perturbation associated with  $v_y$  and  $v_z$  as the curl of the vector potential  $\underline{e}_x \delta A(y, z)$ ,

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\* This excludes the interchange mode discussed in Appendix III, which involves a wave vector principally in the  $xz$ -plane.

where  $\underline{e}_x$  represents a unit vector in the  $x$ -direction. The hydromagnetic equation for  $\delta A$  becomes

$$\frac{\partial \delta A}{\partial t} = -v_z B(z). \quad (\text{IV2})$$

If the thermal gas density and pressure perturbations are  $\delta \rho$  and  $\delta p$ , and if the cosmic ray gas pressure perturbation is  $\delta P$ , then the linearized equations of motion are

$$\rho \frac{\partial v_y}{\partial t} = -\frac{\partial \delta p}{\partial y} - \frac{\partial \delta P}{\partial y} - \frac{1}{4\pi} \frac{dB}{dz} \frac{\partial \delta A}{\partial y}, \quad (\text{IV3})$$

$$\rho \frac{\partial v_z}{\partial t} = -\frac{\partial \delta p}{\partial z} - \frac{\partial \delta P}{\partial z} - \frac{B}{4\pi} \nabla^2 \delta A - \frac{1}{4\pi} \frac{dB}{dz} \frac{\partial \delta A}{\partial z} - g \delta \rho, \quad (\text{IV4})$$

$$\frac{\partial \delta \rho}{\partial t} + v_z \frac{d\rho}{dz} + \rho \left( \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) = 0, \quad (\text{IV5})$$

$$\frac{\partial \delta p}{\partial t} + v_z \frac{dp}{dz} + \gamma v_z \rho \left( \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) = 0, \quad (\text{IV6})$$

assuming that the pressure variation  $\delta p$  in a given element of gas varies as

$$\frac{\delta p}{p} = \gamma \frac{\delta \rho}{\rho},$$

where  $\gamma$  is a constant.

The cosmic ray pressure perturbation is readily computed from the fact that the cosmic ray gas is not significantly affected by gravity and the speed of sound in the cosmic ray gas is very much larger than any of the wave velocities to be considered here. Hence the cosmic ray gas pressure is uniform along any given line of force. The volume of a tube of force does not vary to first order in the perturbation, so to the order considered,  $d\delta P/dt = 0$ , i.e.

$$\frac{\partial \delta P}{\partial t} + v_z \frac{dP}{dz} = 0. \quad (\text{IV7})$$

To solve these equations solve (IV2) for  $v_z$  and substitute into (IV7). Integrating the result over time gives

$$\delta P = \frac{\beta u^2}{B} \frac{d\rho}{dz} \delta A. \quad (\text{IV8})$$

Differentiate (IV3) with respect to time and use (IV6) and (IV8) to eliminate  $\delta \rho$  and  $\delta P$ . The result may be written

$$Q^2 v_z = -\frac{u^2}{B} \left[ \frac{1}{\rho} \frac{d\rho}{dz} \left( 1 + \alpha + \beta - \frac{\gamma}{2} \right) + \gamma \frac{\partial}{\partial z} \right] \frac{\partial^2 \delta A}{\partial t \partial y}, \quad (\text{IV9})$$

where  $Q^2$  is the acoustic wave operator

$$Q^2 \equiv \frac{\partial^2}{\partial t^2} - \gamma u^2 \frac{\partial^2}{\partial y^2}.$$

It is now possible to return to the equation (IV5) and (IV6) for  $\delta\rho$  and  $\delta p$  and employ (IV2) to eliminate  $v_z$  with (IV9) to eliminate  $v_y$ .

The results can be integrated once over time yielding

$$Q^2 \frac{\delta\rho}{\rho} = \frac{1}{B} \left\{ \left( \frac{1}{2\rho} \frac{d\rho}{dz} + \frac{\partial}{\partial z} \right) Q^2 \delta A + v^2 \left[ \left( 1 + \alpha + \beta - \frac{\gamma}{2} \right) \frac{1}{\rho} \frac{d\rho}{dz} + \gamma \frac{\partial}{\partial z} \right] \frac{\partial^2 \delta A}{\partial y^2} \right\}, \quad (IV10)$$

$$Q^2 \frac{\delta p}{p} = \frac{1}{B} \left\{ \left[ \frac{(1 - \gamma/2)}{\rho} \frac{d\rho}{dz} + \gamma \frac{\partial}{\partial z} \right] Q^2 \delta A + \gamma v^2 \left[ \left( 1 + \alpha + \beta - \frac{\gamma}{2} \right) \frac{1}{\rho} \frac{d\rho}{dz} + \gamma \frac{\partial}{\partial z} \right] \frac{\partial^2 \delta A}{\partial y^2} \right\}. \quad (IV11)$$

The final step is to operate on (IV4) with  $Q^2$  and eliminate  $v_x$  with the aid of (IV2), and  $\delta\rho$  and  $\delta p$  with the aid of (IV10) and (IV11). The result can be written

$$\begin{aligned} & v^2 \left[ (\gamma + 2\alpha) Q^2 + \gamma^2 v^2 \frac{\partial^2}{\partial y^2} \right] \frac{\partial^2 \delta A}{\partial z^2} \\ & + \left\{ v^2 \left[ 2\alpha \frac{\partial^2}{\partial y^2} - \frac{g^2}{2v^4} \frac{(\alpha + \gamma/2)}{(1 + \alpha + \beta)^2} \right] Q^2 \right. \\ & \left. - \frac{g^2 (1 + \alpha + \beta - \gamma/2)^2}{(1 + \alpha + \beta)^2} \frac{\partial^2}{\partial y^2} \right\} \delta A - Q^2 \frac{\partial^2 \delta A}{\partial t^2} = 0 \end{aligned} \quad (IV12)$$

after collecting terms and using the equilibrium conditions to eliminate  $dp/dz$ .

Now suppose that  $\delta A$  is of the form

$$\delta A = f(\xi) \exp(i\omega t + iky) \quad (\text{IV}13)$$

where  $\xi = kz$ . The equation reduces to

$$\left[ 2\alpha\gamma - \frac{\omega^2}{k^2 u^2} (\gamma + 2\alpha) \right] \frac{d^2 f}{d\xi^2} + \left\{ \frac{(1 + \alpha + \beta - \gamma)(1 + \alpha + \beta) - \alpha\gamma/2}{k^2 L^2} \right. \\ \left. - 2\alpha\gamma + \frac{\omega^2}{k^2 u^2} (2\alpha + \gamma) \left( 1 + \frac{1}{4k^2 L^2} \right) - \left( \frac{\omega^2}{k^2 u^2} \right)^2 \right\} f = 0 \quad (\text{IV}14)$$

where  $L$  is the scale height  $g/u^2(1 + \alpha + \beta)$ . We impose the boundary condition that the perturbation  $\delta A$  vanish at the base of the atmosphere  $z = 0$  and remain finite as  $z \rightarrow +\infty$ . In order to obtain solutions which satisfy both conditions it is necessary that the coefficients of  $f$  and  $d^2 f/d\xi^2$  have the same sign. Hence unstable solutions ( $k$  real and  $\omega = -i/\tau$  where  $\tau > 0$ ) occur provided only that

$$\frac{(1 + \alpha + \beta - \gamma)(1 + \alpha + \beta) - \alpha\gamma/2}{k^2 L^2} > 2\alpha\gamma + u^2 (2\alpha + \gamma) \left( 1 + \frac{1}{4k^2 L^2} \right) + u^4, \quad (\text{IV}15)$$

where  $U = 1/k\omega\tau$ . For marginal stability ( $U = 0$ ) there are solutions provided only that

$$Y > 2\alpha\gamma k^2 L^2 \quad (\text{IV}16)$$

where  $Y \equiv (1+\alpha+\beta)(1+\alpha+\beta-\gamma) - \alpha\gamma/2$ . Thus instability occurs first at long wavelengths  $kL \rightarrow 0$ , and requires only that  $Y > 0$ . The result is simply interpreted. If  $\gamma > 1$  in the absence of magnetic field and cosmic rays ( $\alpha = \beta = 0$ ), there are no solutions with marginal instability. The isothermal atmosphere is stable. The effect of the field and/or the cosmic rays is instability, so that if  $\alpha$  and  $\beta$  are sufficiently large, they drive the system to instability no matter how stable the thermal gas might be itself. As mentioned in the text, and in Appendix II, the effective  $\gamma$  for the interstellar thermal gas is one or less so that there is instability at long wavelengths for any  $\alpha, \beta > 0$ .

The next question is whether the instability grows sufficiently rapidly to be a significant effect. To demonstrate the growth rate write  $\alpha = 1/k^2 L^2$ ,  $n = L/\omega\tau$ , and  $U = n\alpha^{1/2}$ , where  $n$  specifies the growth rate in terms of the time required to move one scale height at a speed  $u$ . Typically  $L = 100$  pc,  $u = 10$  km/sec, so that if  $\tau \leq 10^8$  years, we must have  $n \geq 0.1$ . The condition (IV15) for instability becomes  $\gamma(\alpha) < 0$ , where

$$y(x) \equiv x^2 \left[ n^4 + n^2 (2\alpha + \gamma)/4 \right] + 2\alpha\gamma + x \left[ n^2 (2\alpha + \gamma) + \alpha\gamma/2 - (1+\alpha+\beta-\gamma)(1+\alpha+\beta) \right]. \quad (\text{IV}18)$$

It is obvious that  $y(x)$  becomes large without limit as  $x \rightarrow \pm\infty$ . \* There is a minimum value of  $y$  at  $x = 2b/a$  where  $b = \gamma - n^2(2\alpha + \gamma)$ ,  $a = n^2(4n^2 + 2\alpha + \gamma)$ . The minimum value of  $y$  is negative, given instability, provided only that

$$b^2 > 2\alpha\gamma a. \quad (\text{IV}19)$$

The condition for marginal stability was  $\gamma > 0$ . For  $n$  as small as 0.1, corresponding to a growth period of  $10^8$  years,  $a$  is of the order of  $10^{-2}$  and  $b \cong \gamma$  so that (IV19) is hardly more than the requirement for marginal stability. The interstellar particle-field system is always unstable under these circumstances because of the apparently low value of  $\gamma$  and the fact that

$$\alpha, \beta > 0.$$

To calculate the maximum growth rate  $n$ , note that when  $(\gamma - 2\alpha)\gamma \ll (\gamma + 2\alpha)(\alpha\gamma + \gamma)$  the maximum value of  $n$  for which (IV19) is satisfied and  $b > 0$  is

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\* Negative values of  $x$  are physically uninteresting because  $k$  is imaginary then. Hence we require that  $b > 0$ .

$$n_{\max} \approx \frac{\gamma}{[2(\gamma + 2\alpha)(\alpha\gamma + \gamma)]^{1/2}}.$$

This maximum  $n$  is associated with infinite wavelength in the vertical direction, so it should not be taken too seriously. The effective maximum  $n$  is somewhat less. For a galactic magnetic field of  $5 \times 10^{-6}$  gauss, a cosmic ray pressure of  $0.45 \times 10^{-12}$  dynes/cm<sup>3</sup>, and a thermal gas density of 3 hydrogen atoms/cm<sup>3</sup> with an rms velocity  $u = 8$  km/sec (see section II), it follows that  $\rho u^2 = 3.0 \times 10^{-12}$  dynes/cm<sup>3</sup>,  $\alpha = 0.3$ , and  $\beta = 0.15$ . Put  $\gamma = 1$ . Then  $\gamma = 0.50$  and the maximum growth rate is  $n = 0.3$ , giving  $\tau \approx 3 \times 10^7$  years. A much weaker galactic field permits equilibrium with one atom/cm<sup>3</sup>, giving  $\rho u^2 = 1 \times 10^{-12}$  dynes/cm<sup>3</sup> with  $\alpha \approx 0$  and  $\beta = 0.45$ . With  $\alpha \approx 0$  the requirement (IV19) becomes  $\gamma > n^2$  for instability, and hence the maximum growth rate is  $n_{\max} = 0.807$ , giving  $\tau = 1.2 \times 10^7$  years.

It is evident that a large number of cases could be investigated, involving various values of  $\alpha, \beta, \gamma$  and employing different boundary conditions, including cutting off the atmosphere at some specified height  $z = h$  with a uniform cosmic ray gas pressure  $P_0$  or a large-scale uniform magnetic field  $B_0$  beyond. The reader who wishes to investigate the possible effects of significant intergalactic pressure may find some interest in this. Stability can be achieved under some circumstances, as in the limit of strong intergalactic magnetic fields. The stability arises from the fact that with an external pressure

the weight of the gas no longer is responsible for confining the magnetic field. It is the weight of the gas on the field which produces the instability.

We shall content ourselves here with two simple examples of the nature of the unstable flow to illustrate the draining of the thermal gas into the low regions along the lines of force. Suppose that the cosmic ray gas is absent and the thermal gas is cold. Then  $\beta = 0$ ,  $u^2 = 0$ ,  $\alpha u^2 = \frac{1}{2} V_A^2 \neq 0$  where  $V_A$  is the Alfven speed  $B / (4\pi\rho)^{1/2}$ . Eqn. (14) reduces to

$$\frac{d^2 f}{dz^2} + K^2 f = 0$$

where, with  $s^2 \equiv V_A^2 \tau^2 / L^2$ ,

$$K^2 = \frac{1}{4} k^2 s^2 \left[ 1 - \frac{4}{s^2} \left( 1 + \frac{4}{k^2 L^2} \right) - \left( \frac{2}{s^2 k^2 L^2} \right)^2 \right]$$

$$\approx \frac{1}{4} k^2 s^2$$

in the limit of long growth times,  $s^2 \gg 1$ . In order that  $\delta A$  vanish at  $z = 0$ , put

$$\delta A = \epsilon B L \exp \frac{t}{\tau} \cos ky \sin Kz,$$

where  $\epsilon \ll 1$ . It follows that the magnetic lines of force are given by

$$z - z_0 = \epsilon L \exp \frac{t}{\tau} \sin K z_0 (1 - \cos k_y)$$

where  $z_0$  is the value of  $z$  at which the line crosses the  $z$ -axis.

It is readily shown from (IV9) that

$$v_1 = - \frac{\epsilon}{2} V_A s k L \exp \frac{t}{\tau} \sin k_y \sin K z$$

and from (IV2) that

$$v_2 = - \epsilon \frac{V_A}{s} \exp \frac{t}{\tau} \cos k_y \sin K z.$$

It is evident that the motion drains the thermal gas away from the high regions

$k_y = \pm (2n+1) \pi$  along the magnetic lines of force into the low regions

$k_y = \pm 2n \pi$ , as illustrated in Fig. 2. When  $s \gg 1$  the motion is principally in the horizontal direction.

Suppose, on the other hand, that the magnetic field is very weak,\* the thermal gas is cold, and most of the pressure is contributed by the cosmic ray gas. Then  $\alpha \cong 0$ ,  $\beta \cong 0$ ,  $\beta \alpha^2 \equiv C^2 \neq 0$ , where  $C$  is the equivalent thermal velocity,  $C = (P/\rho)^{1/2}$ . The atmosphere is unstable for any value of  $C > 0$ . Eqn. (14) reduces to

---

\*The magnetic field is taken to be so weak that its pressure can be neglected but not so weak that the radius of gyration of the cosmic ray particles exceeds the scale  $L$  of the system. The range  $B = 10^{-6} - 10^{-9}$  gauss would be a reasonable compromise in this respect.

$$\beta^2 \left[ \frac{1}{k^2 L^2} - \left( \frac{1}{k^2 C^2 \tau^2} \right)^2 \right] f = 0.$$

Thus, the nontrivial unstable solution is  $k^2 C^2 \tau^2 = k L$  with  $f$  any arbitrary function of  $z$ , provided only that  $f$  is single valued and continuous so as not to violate the assumptions which went into the initial linearization of the equations. The reason for the arbitrariness of  $f$  is that neither the thermal gas nor the vanishing magnetic field can resist compression, and the cosmic ray gas avoids compression by redistributing itself along the magnetic lines of force.

Put

$$\delta A = \epsilon L \exp \frac{t}{\tau} \cos k y f(z)$$

and suppose that  $df/dz = O(kf)$  for all values of  $z$ . Then the magnetic lines of force are given by

$$z - z_0 = \epsilon L \exp \frac{t}{\tau} (1 - \cos k y) f(z_0),$$

and

$$v_1 = -\epsilon \frac{L}{\tau} \exp \frac{t}{\tau} \sin k y f(z),$$

$$v_2 = -\epsilon \frac{L}{\tau} \exp \frac{t}{\tau} \cos k y f(z).$$

Thus again the motion represents a draining of the thermal gas along the magnetic lines of force from the high regions into the low regions.

## Appendix V. Stability of a Circular Geometry

Consider the stability of a two dimensional atmosphere with circular symmetry, involving a magnetic field  $B(\varpi)$  whose lines of force form concentric circles about the origin. Here  $\varpi$  represents distance measured from the origin and  $\phi$  represents azimuth measured around the origin. The gravitational field is radially inward with magnitude  $g(\varpi)$ . The thermal gas is isothermal with rms thermal velocity  $u$  in any given direction, so that  $p = \rho u^2$ . The cosmic ray gas pressure is  $P$ . Equilibrium requires that

$$\frac{d}{d\varpi} \left( \rho + \frac{B^2}{8\pi} + P \right) + \frac{B^2}{4\pi\varpi} = -\rho(\varpi) g(\varpi). \quad (\nabla 1)$$

The simple case that the thermal gas, the magnetic field, and the cosmic ray gas pressures are in a fixed ratio, defined by eqn. (2) in the text, permits (V1) to be written

$$\frac{1}{\rho} \frac{d\rho}{d\varpi} = - \frac{g(\varpi)}{u^2(1+\alpha+\beta)} - \frac{2\alpha}{(1+\alpha+\beta)\varpi} = - \frac{1}{L}. \quad (\nabla 2)$$

Introduce a transverse perturbation  $v_\phi$ ,  $V_\phi$  with the magnetic perturbation described by the vector potential  $\underline{e}_\phi \delta A$ . The linearized perturbation equations may then be written

$$\frac{\partial \delta A}{\partial t} = v_\phi B(\varpi), \quad (\nabla 3)$$

$$\rho \frac{\partial v_{\theta}}{\partial t} = - \frac{\partial}{\partial \omega} (\epsilon p + \epsilon P) + \frac{B^2}{4\pi} \nabla^2 \delta A + \frac{1}{4\pi \omega} \frac{d}{d\omega} (\omega B) \frac{\partial \delta A}{\partial \omega} - g \epsilon \rho, \quad (\text{V4})$$

$$\rho \frac{\partial v_{\phi}}{\partial t} = - \frac{1}{\omega} \frac{\partial}{\partial \phi} (\epsilon p + \epsilon P) + \frac{1}{4\pi \omega} \frac{d}{d\omega} (\omega B) \frac{1}{\omega} \frac{\partial \delta A}{\partial \phi}, \quad (\text{V5})$$

$$\frac{\partial \epsilon p}{\partial t} + v_{\omega} \frac{dp}{d\omega} + \rho \left[ \frac{1}{\omega} \frac{\partial}{\partial \omega} (\omega v_{\omega}) + \frac{1}{\omega} \frac{\partial v_{\phi}}{\partial \phi} \right] = 0, \quad (\text{V6})$$

$$\frac{\partial \epsilon p}{\partial t} + v_{\omega} \frac{dp}{d\omega} + \gamma_0 \rho \left[ \frac{1}{\omega} \frac{\partial}{\partial \omega} (\omega v_{\omega}) + \frac{1}{\omega} \frac{\partial v_{\phi}}{\partial \phi} \right] = 0, \quad (\text{V7})$$

$$\frac{\partial \epsilon P}{\partial t} + v_{\omega} \frac{dP}{d\omega} = 0. \quad (\text{V8})$$

The general solution of these equations is difficult because of the radial dependence. We know that the thermal gas is stable by itself if  $\gamma > 1$ . The question is whether the magnetic field and the cosmic ray gas tend to produce instability in this configuration, as they did in the horizontal field of Appendix IV. So it is sufficient to consider the thermal gas to be cold,  $v^2 = 0$ , thereby exposing the basic instabilities, if any, of the magnetic field and cosmic ray gas.

Consider first the stability of the magnetic field. Put  $\beta = 0$

and  $\alpha v^2 = \frac{1}{2} V_A^2 \neq 0$  where  $V_A$  is the Alfven speed  $B/(4\pi\rho)^{1/2}$

The quantities  $\rho$ ,  $P$ ,  $\delta\rho$ , and  $\delta P$  vanish in the equations. Divide

(V6) by  $\rho$  and differentiate with respect to time. Use (V3) to eliminate  $V_\varpi$

and (V5) to eliminate  $v_\phi$  from the result, obtaining

$$\frac{\partial^2}{\partial t^2} \frac{\delta\rho}{\rho} = -\frac{1}{B} \left[ \left( \frac{1}{\varpi} + \frac{1}{\rho} \frac{d\rho}{d\varpi} \right) \left( \frac{\partial^2}{\partial t^2} + \frac{V_A^2}{\varpi^2} \frac{\partial^2}{\partial \phi^2} \right) + \frac{\partial^3}{\partial \varpi \partial t^2} \right] \delta A.$$

Then divide (V4) by  $\rho$  and differentiate twice with respect to time. After

eliminating  $\delta\rho/\rho$  the result can be written

$$\left[ \left( \frac{\partial^2}{\partial t^2} - V_A^2 \nabla^2 \right) \frac{\partial^2}{\partial t^2} + \frac{g^2}{V_A^2} \left( \frac{\partial^2}{\partial t^2} + \frac{V_A^2}{\varpi^2} \frac{\partial^2}{\partial \phi^2} \right) \right] \delta A = 0 \quad (\text{V9})$$

with the aid of the equilibrium relation (V2), which reduces to

$$\frac{1}{2\rho} \frac{d\rho}{d\varpi} + \frac{1}{\varpi} = - \frac{g}{V_A^2}$$

in the present special circumstances. A solution of the form

$$\delta A = R(\varpi) \exp i(\omega t + n\phi) \quad (\text{V10})$$

leads to

$$\frac{d^2 R}{d\varpi^2} + \frac{1}{\varpi} \frac{dR}{d\varpi} + \left[ \frac{\omega^2}{V_A^2} - \frac{g^2}{V_A^4} - \left( 1 + \frac{g^2}{V_A^2 \omega^2} \right) \frac{n^2}{\varpi^2} \right] R = 0. \quad (\text{V11})$$

The differential equation for the radial dependence is easily solved for the cases that

$g$  is independent of  $\varpi$ , in which case the system is bound in a conical potential well, or  $g$  increases proportional to  $\varpi$ , so that the system is bound in a parabolic potential well. To take the last case first, put

$$g(\varpi) = g(a) \frac{\varpi}{a} \quad (\text{V12})$$

where  $g(a)$  is the gravitational acceleration of  $\varpi = a$ . Then at small  $\varpi$ , such that  $\varpi \ll V_A \tau$ , the term  $g^2/V_A^2$  may be neglected compared to  $\omega^2/V_A^2$  and the solution is

$$R(\varpi) = J_n(q\varpi) \quad (\text{V13})$$

where

$$q = \frac{1}{V_A \tau} \left[ \left( 2n \frac{\frac{1}{2} g(a) \tau^2}{a} \right)^2 - 1 \right]^{1/2}. \quad (\text{V14})$$

In the limit of large  $\tau$ ,

$$q\varpi \sim n \frac{g(a)\tau}{V_A} \frac{\varpi}{a}, \quad (\text{V15})$$

which may be large compared to one while  $\omega \ll V_A \tau$ . It is evident that the solution is well behaved, satisfying  $\delta A = 0$  at some inner radius  $\omega = b$ , say, and oscillating with declining amplitude with increasing  $\omega$ . Within the limitations of these boundary conditions, the system is unstable, just as in the flat atmosphere considered in Appendix IV.

If  $g$  is independent of  $\omega$  \* the solution of the radial equation is

$$R(\omega) = Z_{ip} \left( i \frac{\omega}{l} \right), \quad (\text{V16})$$

where  $Z$  represents a Bessel function and

$$\frac{1}{l} = \left( \frac{g^2}{V_A^4} + \frac{1}{V_A^2 \tau^2} \right)^{1/2}, \quad p = \pm n \left( \frac{g^2 \tau^2}{V_A^2} - 1 \right)^{1/2}. \quad (\text{V17})$$

In the limit of large  $\tau$ ,  $l \sim V_A^2/g$  and  $p \sim ng\tau/V_A$ . The solution

$$R(\omega) = i \left[ J_{ip}(\omega/l) - \exp(-\pi p) J_{-ip}(\omega/l) \right] \quad (\text{V18})$$

is real and well behaved at all finite  $\omega$ , going to zero as  $\omega \rightarrow \infty$ .

This is readily seen from the leading terms of the expansion at small  $\omega/l$ ,

---

\* There is a discontinuity in the gravitational field at the origin in this case.

$$R(\omega) \cong \frac{\exp(-\pi\rho/2)}{\rho} \left\{ \frac{\exp(i\rho \ln \omega/2l)}{\Gamma(i\rho)} + \frac{\exp(-i\rho \ln \omega/2l)}{\Gamma(-i\rho)} \right\} \quad (\text{V19})$$

which oscillates rapidly, and from the asymptotic expansion

$$R(\omega) \sim \exp\left(\frac{\pi\rho}{2}\right) \left[1 - \exp(-2\pi\rho)\right] \left(\frac{l}{2\pi\omega}\right)^{1/2} \exp\left(-\frac{\pi}{l}\right) \quad (\text{V20})$$

at large  $\omega/l$ . Thus again the system is unstable for large  $\tau$  ( $g\tau \gg v_A$ ).

Consider the other example now in which the magnetic field pressure can be neglected but the cosmic ray gas pressure is finite. Then  $\delta A = 0$ ,

$\delta p = 0$ . Differentiate (V6) with respect to time and use (V4) and (V5) to eliminate  $v_\omega$  and  $v_\rho$ , solving the result for  $\delta p$ , which may then be used to eliminate  $\delta p$  from the previous equation. The resulting equation is

$$\frac{\partial^4 \delta P}{\partial t^4} + \left(\frac{dg}{d\omega} - \frac{g}{\omega}\right) \frac{\partial^2 \delta P}{\partial t^2} + \frac{g^2}{\omega^2} \frac{\partial^2 \delta P}{\partial \theta^2} = 0. \quad (\text{V21})$$

It is interesting to note that this equation is valid whenever  $dP/d\omega = -\rho g$  and does not depend upon  $P \propto \rho$  as a function of  $\omega$ .

Note that the coefficients in (21) are functions only of  $\omega$ , whereas the derivative of  $\delta P$  with respect to  $\omega$  does not appear in the equation. Thus if we suppose that  $\delta P$  can be written

Differentiate (V8) with respect to time and use (V4) to eliminate  $v_\omega$

$$\delta P = G(\varpi) \exp i(\omega t + n\phi) \quad (\text{V22})$$

we find that  $G(\varpi)$  is an arbitrary function of  $\varpi$ , so long as it is single valued, etc. for the same reason that in Appendix IV  $f$  was an arbitrary function when  $\alpha = v^2 = 0$ . We find also that the variables are separable, as assumed in (22), if and only if  $g(\varpi)$  satisfies

$$\omega^4 - \omega^2 \left( \frac{d^2 g}{d\varpi^2} - \frac{g}{\varpi^2} \right) - \frac{n^2 g^2}{\varpi^2} = 0. \quad (\text{V23})$$

The general solution of (23) is

$$g(\varpi) = \frac{\omega^2}{n} \varpi \left( \frac{\varpi^{2n} - c^{2n}}{\varpi^{2n} + c^{2n}} \right), \quad (\text{V24})$$

where  $c$  is an arbitrary constant. If  $c \rightarrow 0, \infty$ , the simple form

$g(\varpi) \propto \varpi$  is obtained, in which the system is bound in a parabolic potential well. Write  $g(\varpi) = g(a) \varpi/a$ . Then for the unstable solution,  $\omega = -i/\tau$ , let  $c \rightarrow \infty$ , yielding the growth time

$$\tau^2 = \frac{a}{n g(a)}.$$

The growth time is thus of the order of the free fall time.

It is evident from these two examples involving the magnetic field and the cosmic ray gas that the system is unstable unless the thermal gas should be so hot and dense with so large a  $\gamma$  as to stabilize the system. This is the same situation as was found in Appendix IV for the flat atmosphere.

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## Figure Captions

Fig. 1. A sketch of the convective interchange of parallel magnetic lines of force in the  $\gamma$ -direction. The circular velocity of the gas and field is indicated by the velocity symbol  $\underline{v}$ .

Fig. 2. A sketch of the local state of the lines of force of the interstellar magnetic field and interstellar gas cloud configuration resulting from the intrinsic instability of a large-scale field along the galactic disk or arm when confined by the weight of the gas.

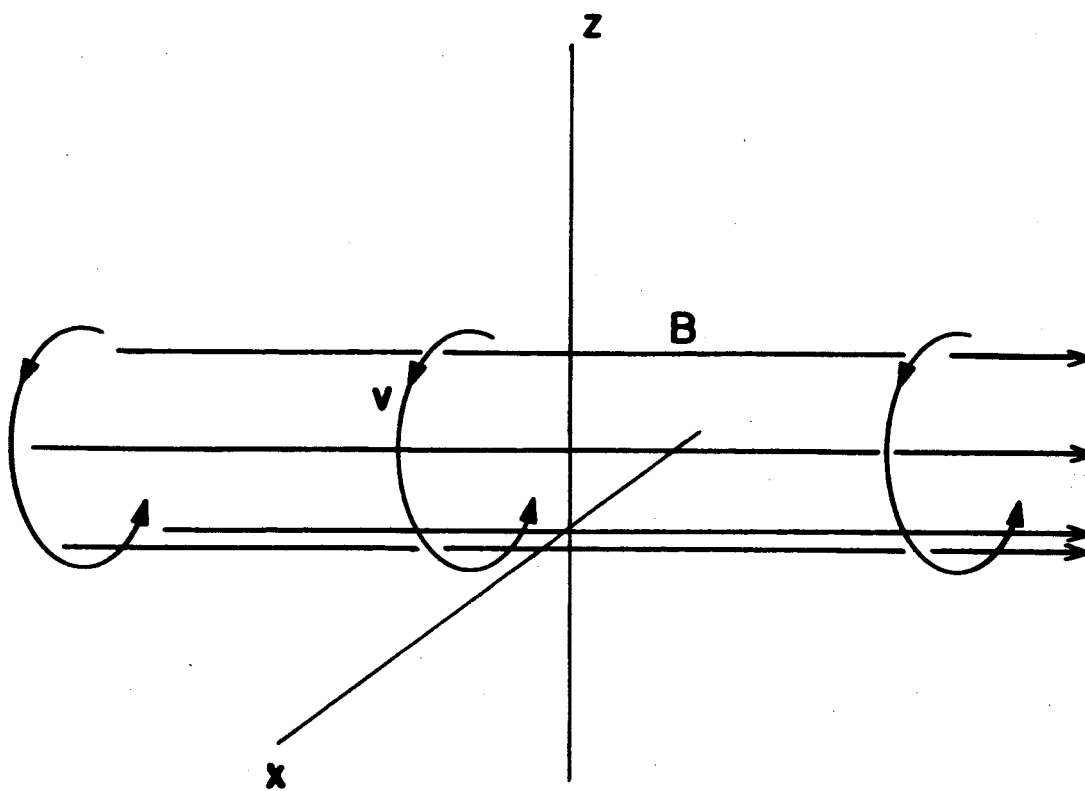


Fig.1

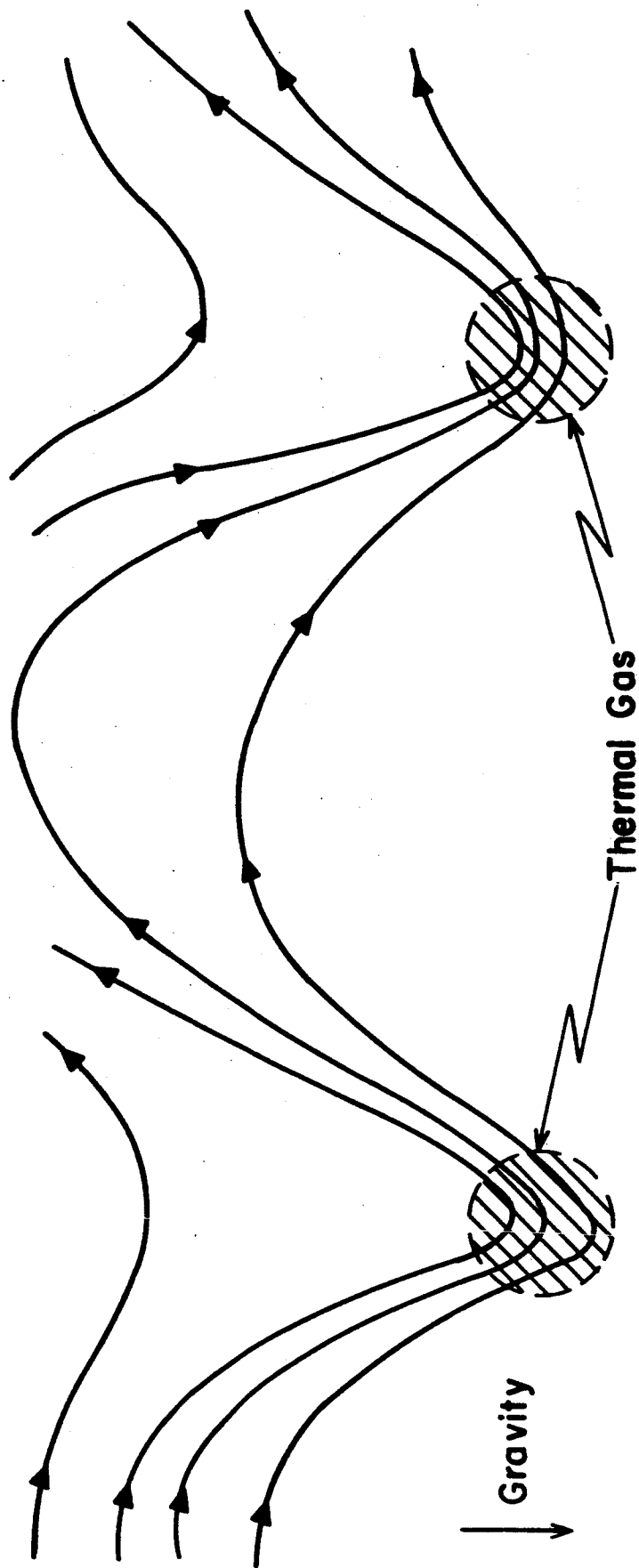


Fig.2